plane, and let $z'_1$, $z'_2$, $z'_3$ represent the corresponding vertices in a $z'$-complex-number-plane.

A transformation can always be found that will change $z_1$ in $z'_1$, and $z_2$ into $z'_2$. For this it is sufficient that $a$ and $b$ in

$$\left.\begin{array}{l} z'_1=az+b \\ z'_2=az_2+b \end{array}\right\} \ldots.(1),$$

shall satisfy the condition

$$a=\frac{z'_2-z'_1}{z_2-z_1}, \text{ and } b=\frac{z'_1z_2-z'_2z_1}{z_2-z_1}.$$

Every transformation of the plane into itself that leaves every figure in the plane similar to itself is expressible in the form

$$z'=az+b\ldots\ (2).$$

The value of $a$ and $b$ in (1) must satisfy the equation $z'_3=az_3+b$.
For this the necessary and sufficient condition is

$$\frac{z'_3-z'_1}{z_3-z_1}=\frac{z'_2-z'_1}{z_2-z_1};\ \therefore \begin{vmatrix} z'_1, & z_1, & 1 \\ z'_2, & z_2, & 1 \\ z'_3, & z_3, & 1 \end{vmatrix}=0.$$

The conditions that the sides shall be equal are

$$\frac{z_1-z_2}{z_2-z_3}=\frac{z_2-z_3}{z_3-z_1}=\frac{z_3-z_1}{z_1-z_2};\ \therefore \begin{vmatrix} z_1, & z_2, & 1 \\ z_2, & z_3, & 1 \\ z_3, & z_1, & 1 \end{vmatrix}=0.$$

Also demonstrated by *WILLIAM HOOVER*, *G. B. M. ZERR*, *J. W. YOUNG*, and *J. SCHEFFER*.

## CALCULUS.

108. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

The hypotenuse of a plane right triangle increases uniformly at the rate of 1-12 of an inch a second. If the legs are as 2 to 3, at what rate is the area of the triangle increasing when the perpendicular from the right angle upon the hypotenuse is 12 inches ?

Solution by C. HORNUNG, A. M., Heidelberg University, Tiffin, O.; H. C. WHITAKER, Ph. D., Manual Training School, Philadelphia, Pa.; J. W. YOUNG, Cornell University, Ithaca, N. Y.; P. S. BERG, Larimore, N. D.; and D. G. DORRANCE, Jr., Camden, N. Y.

Let $x$=the length of the hypotenuse, and $y$=the area.

Then $dx=\frac{1}{12}$, and $dy$ is required when the perpendicular from the right angle to the hypotenuse equals 12.

From the condition of the problem $y=\dfrac{3x^2}{13}$, whence $dy=\dfrac{6xdx}{13}$.

Now when the perpendicular from the right angle to the hypotenuse is 12 the hypotenuse must be 26. Hence, substituting in the last equation we have $dy=\frac{6\times26\times\frac{1}{12}}{13}=1$ ; *i. e.*, the area, at the time mentioned, is increasing at the rate of 1 square inch a second.

Also solved by *L. B. FILLMAN*, *ALOIS F. KOVARIK*, *J. SCHEFFER*, *C. D. SCHMITT*, and *G. B. M. ZERR*.

**109. Proposed by M. E. GRABER, Heidelberg University, Tiffin, Ohio.**

Find the curve in which the product of the perpendiculars drawn from two fixed points to any tangent is constant.

**Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn., and the PROPOSER.**

Let the equation of the tangent be

$$y-y'=\frac{dy'}{dx'}(x-x').$$

And let the two points be $(a,\ 0)$ and $(-a,\ 0)$.

The product of the two perpendiculars is easily found to be

$$\frac{(xdy-ydx)^2-(ady)^2}{(dx)^2+(dy)^2}\text{ which}=b^2,$$

$$\text{or }\frac{(xp-y)^2-(ap)^2}{1+p^2}=b^2,\text{ or }(xp-y)^2=p^2(a^2+b^2)+b^2,$$

$$\text{or }y=px\pm\sqrt{[p^2(a^2+b^2)+b^2]}.$$

This is Clairaut's form, so that we have

$$y=mx\pm\sqrt{[m^2(a^2+b^2)+b^2]},$$

which is the well-known tangent to an ellipse.

Also solved by *H. C. WHITAKER*, and *G. B. M. ZERR*.

## MECHANICS.

**108. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.**

Can it be shown, as a result of Kepler's third law, that the distances are inversely proportional to the squares of the velocities ?

**Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.**

This can be demonstrated for circular orbits as follows :